

Calculus for Everyone

Continuity and One-sided Limits



Please contact Mary.Lester@arkansas.gov,
if you would like the original PowerPoint slides

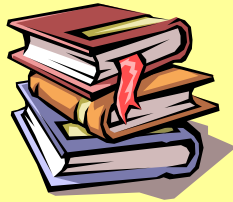
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Calculus for Everyone

**These are template slides to be
used interactively with teachers
and
students filling in answers**



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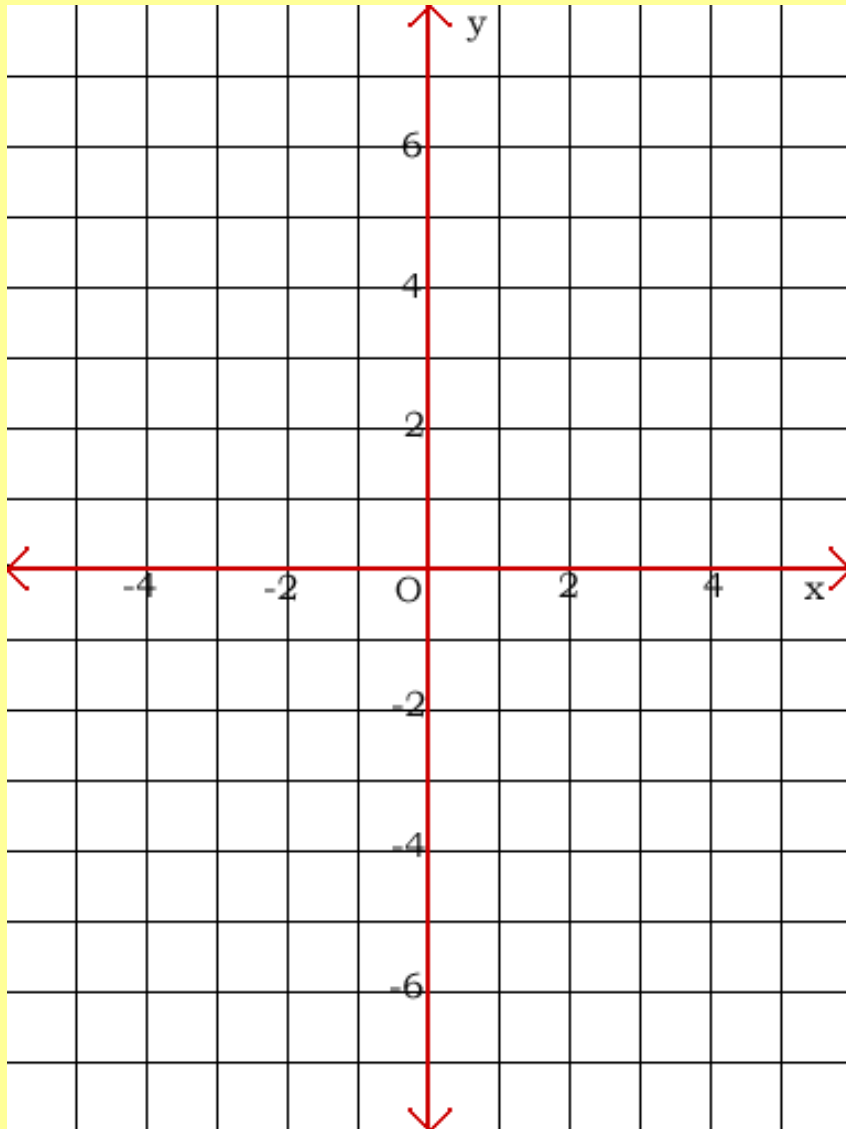
What does it mean to be continuous?

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**“That you can draw the graph
without picking up your pencil”**

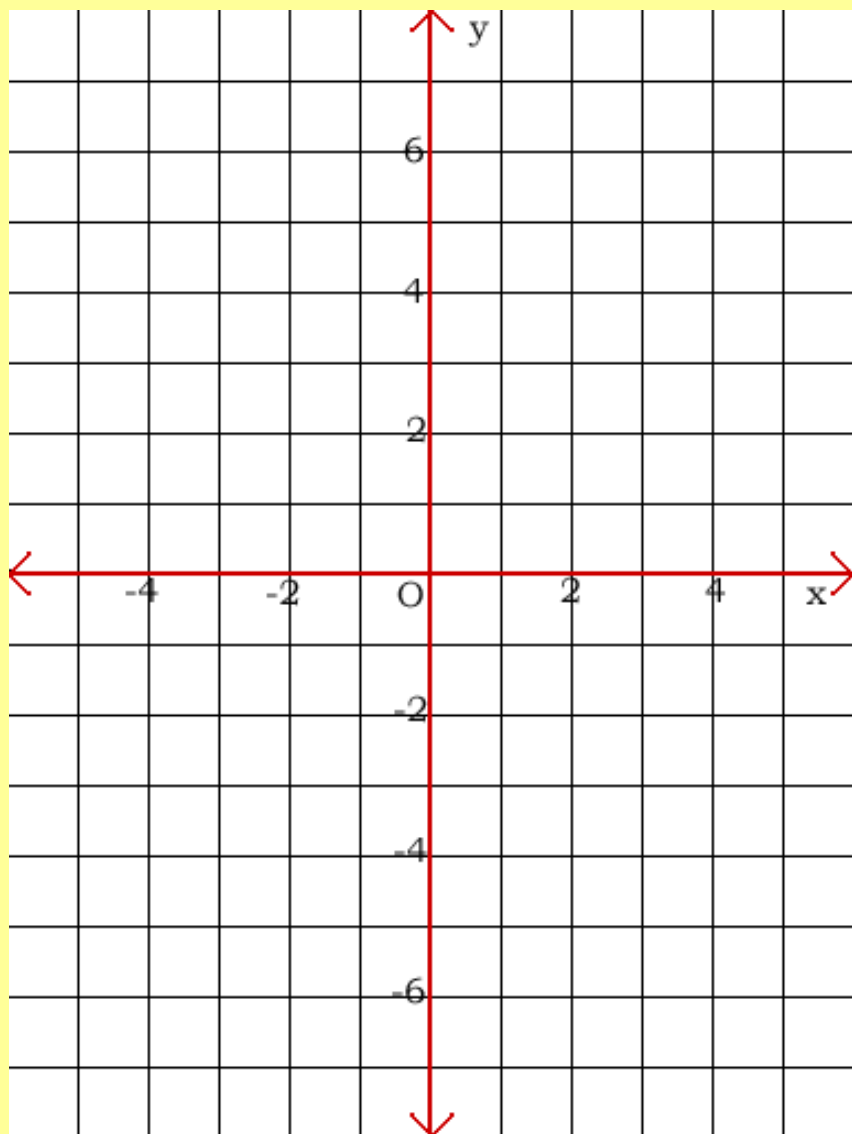
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**Draw $f(x)$
where it is Not
Continuous at "C"**

$f(x)$ is not defined at $x=c$

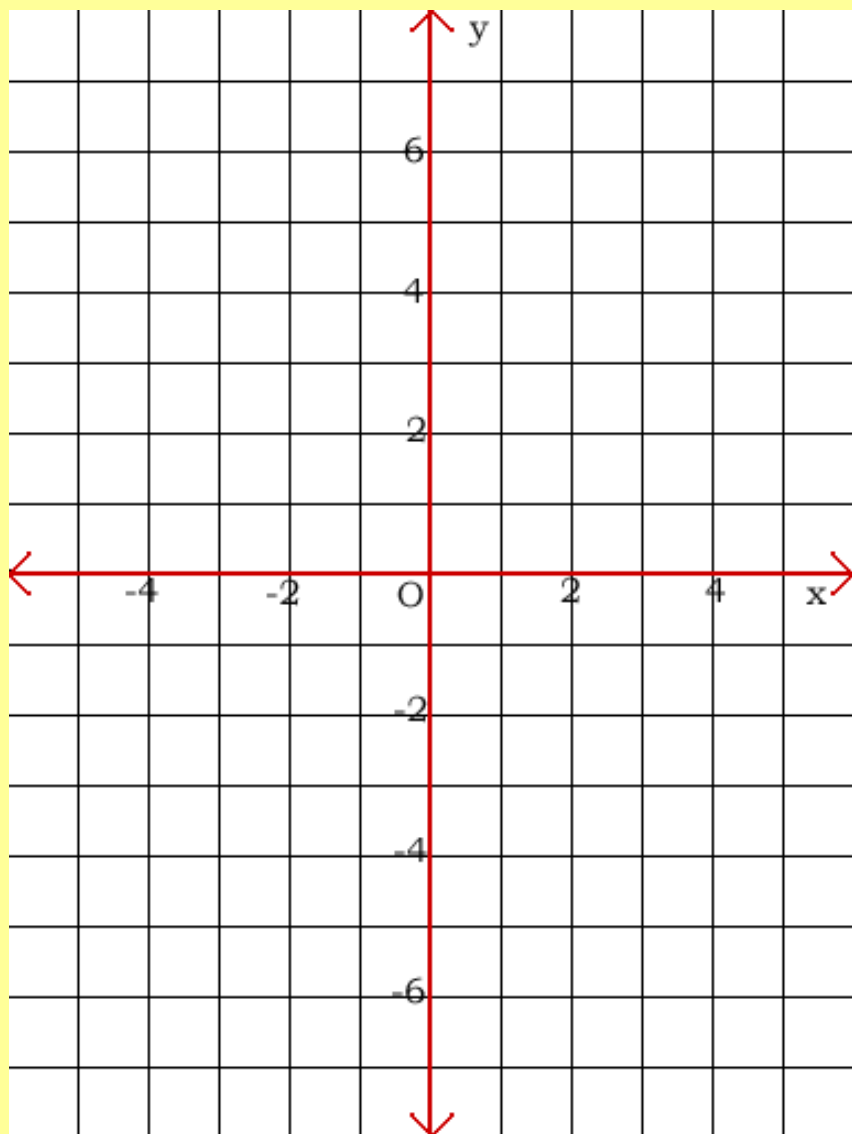
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**Draw $f(x)$ where it is
Not Continuous at "C"**

limit does not exist at
 $x=C$

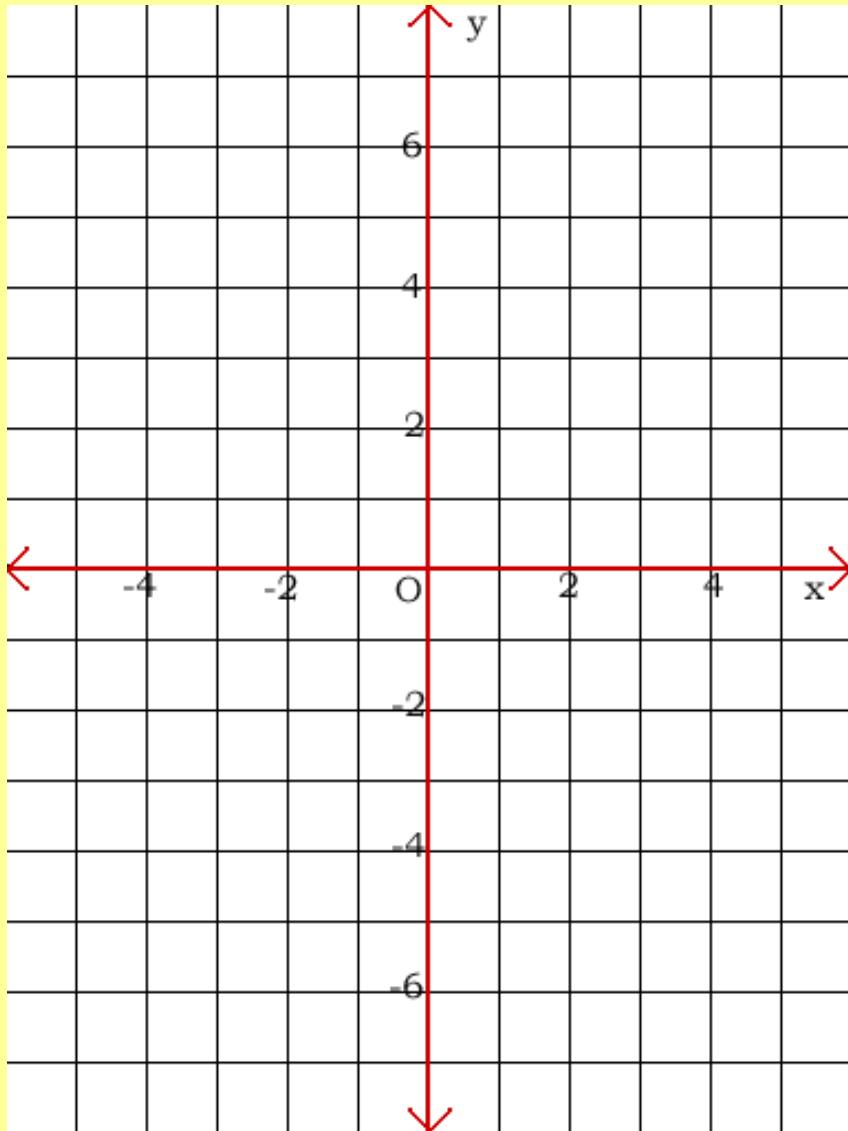
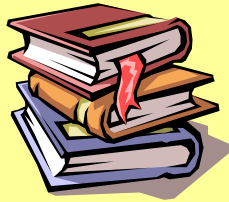
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**Draw $f(x)$ where it is
Not Continuous at " c "**

limit does exist at $x=c$,
but
is not equal to $f(c)$

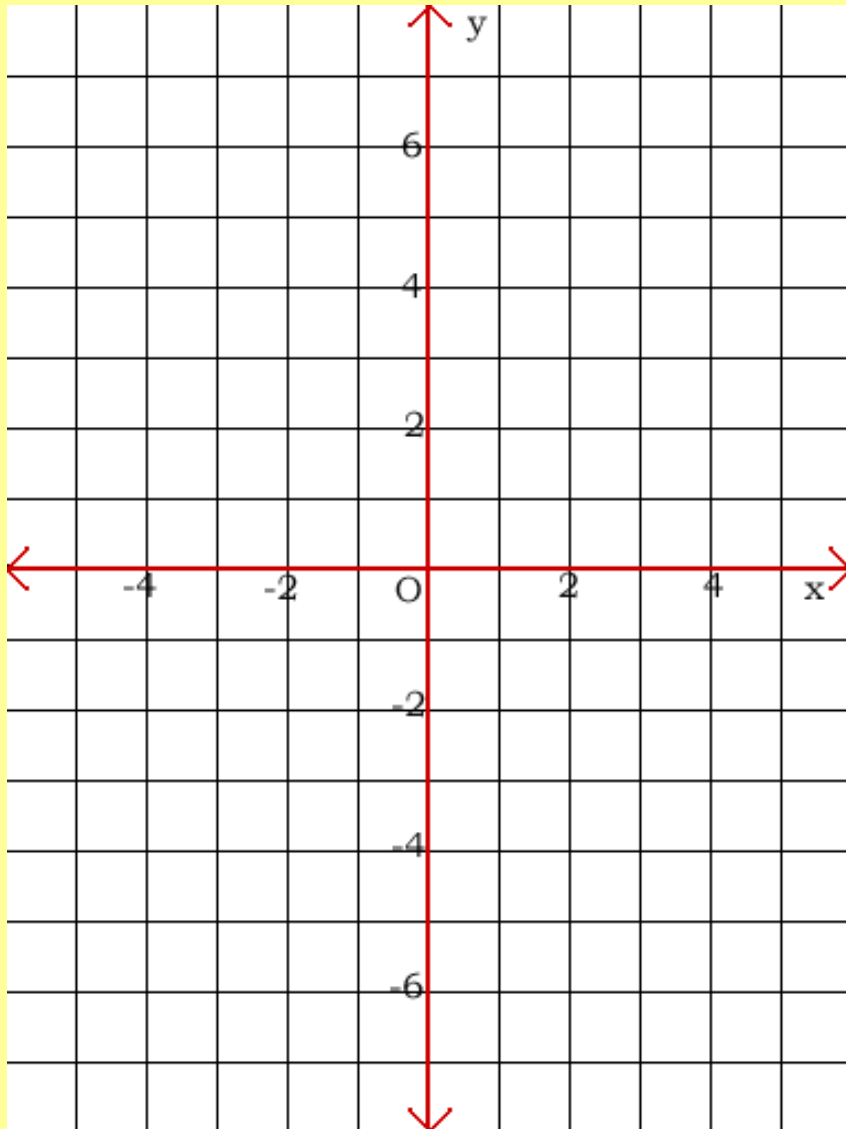
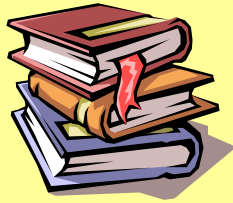
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Definition of Continuity (must know)

- 1.
- 2.
- 3.

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Continuous on an Open Interval (a,b)

A function is **continuous** on an open interval (a,b) if it is continuous at each point on the interval

What does that look like?

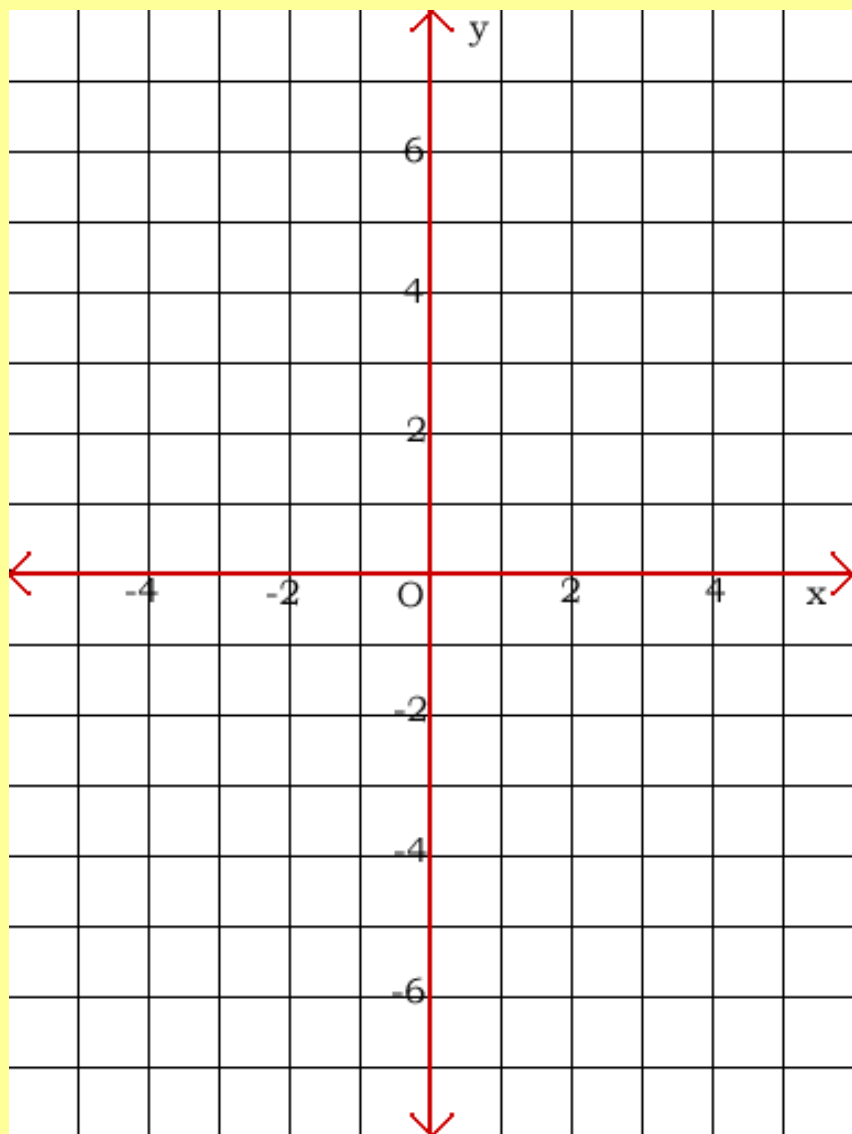
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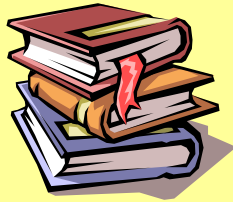
Everywhere Continuous

A function that is continuous over the entire real number line is **everywhere continuous**

What does that look like?



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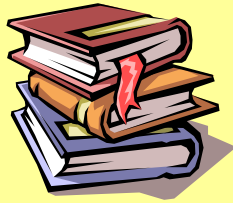
**Discontinuities fall into two categories:
removable and non-removable**

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**Removable discontinuity can
be fixed -- "plug up the hole"**

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**Non-removable discontinuity
can't be "fixed"—there are breaks, asymptotes**

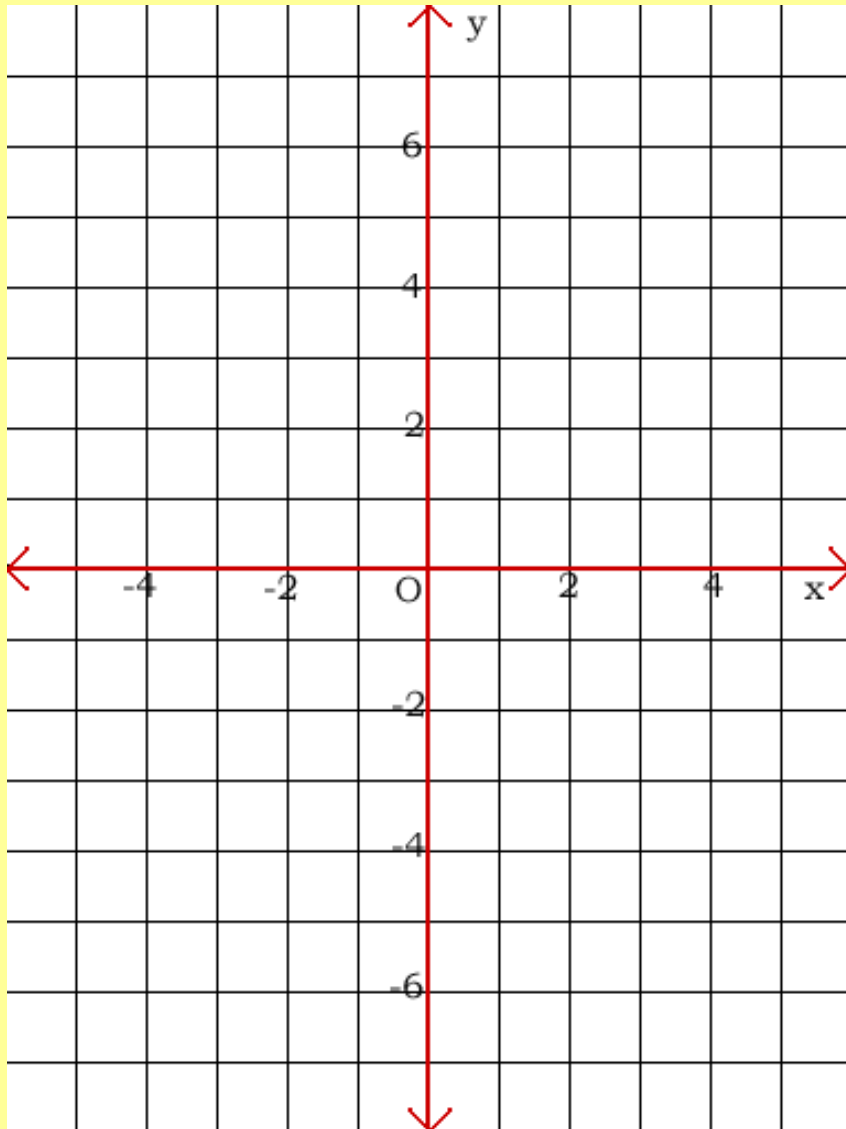
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Continuity of a Function

Removable discontinuity

What does that look like?



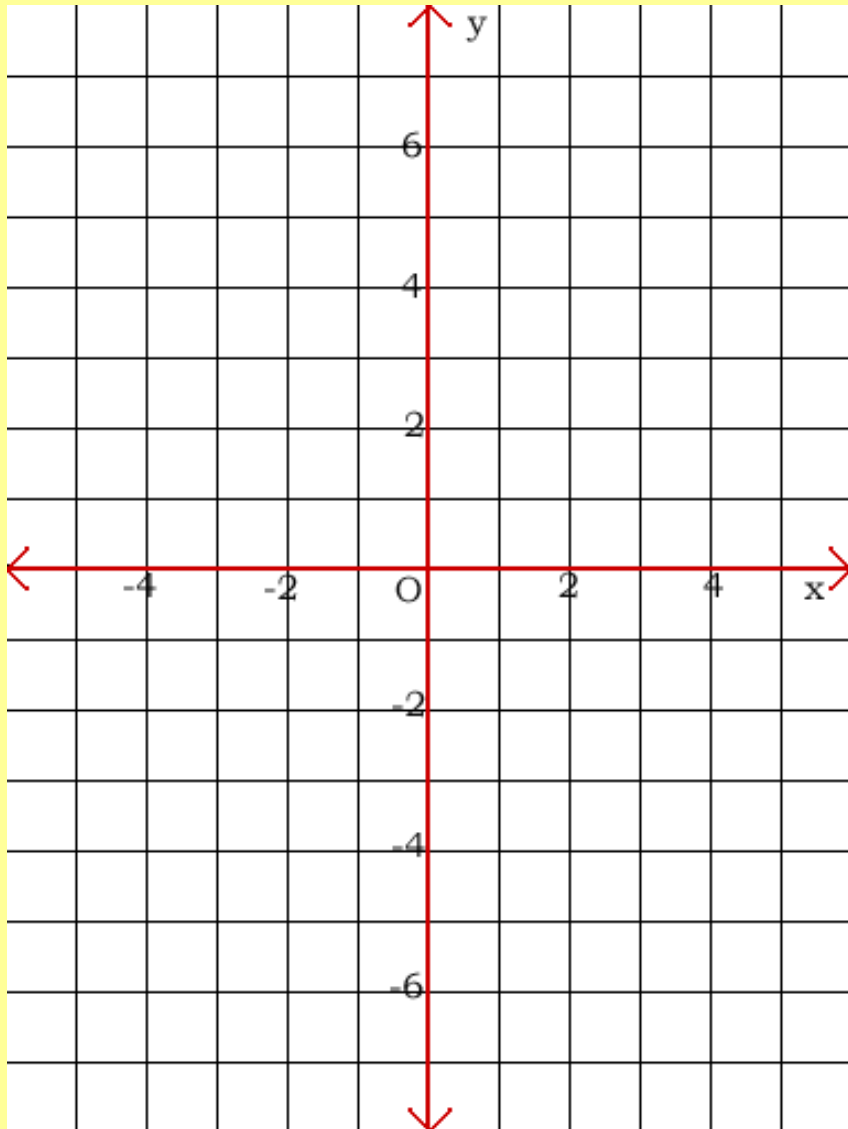
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Continuity of a Function

Non-removable discontinuity

What does that look like?

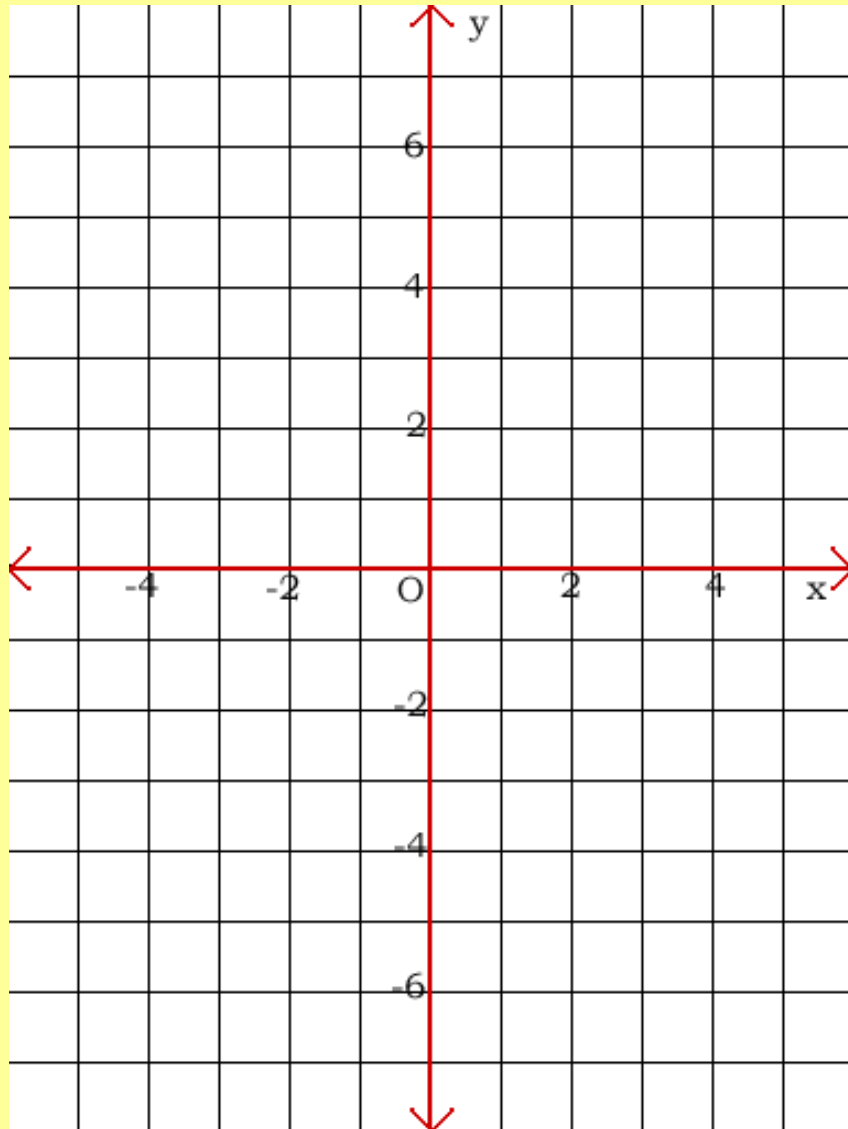


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Ex1 Continuity of a Function

Discuss the continuity of each function



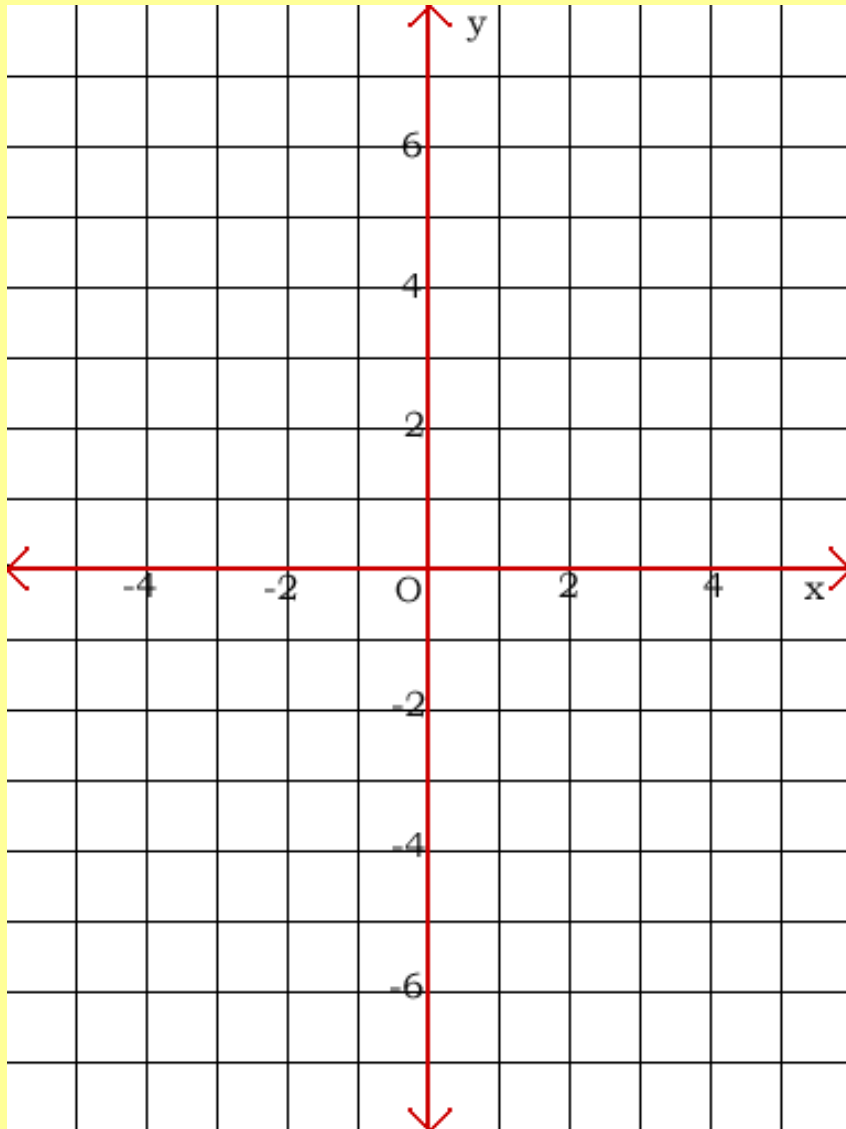
$$f(x) = \frac{1}{x}$$

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Ex1 Continuity of a Function

Discuss the continuity of each function



$$g(x) = \frac{x^2 - 1}{x - 1}$$

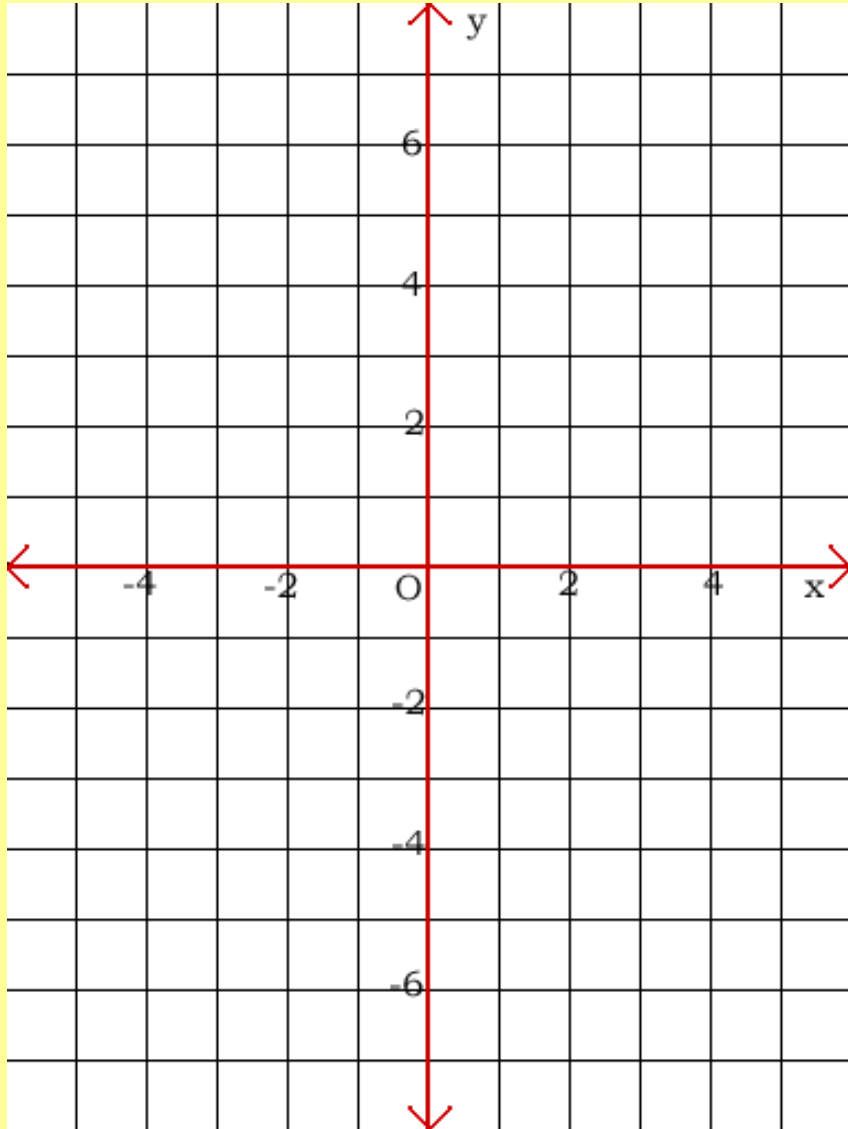
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Ex1 Continuity of a Function

Discuss the continuity of each function

$$h(x) = \begin{cases} x - 1, & x \leq 0 \\ x^2 + 1, & x > 0 \end{cases}$$

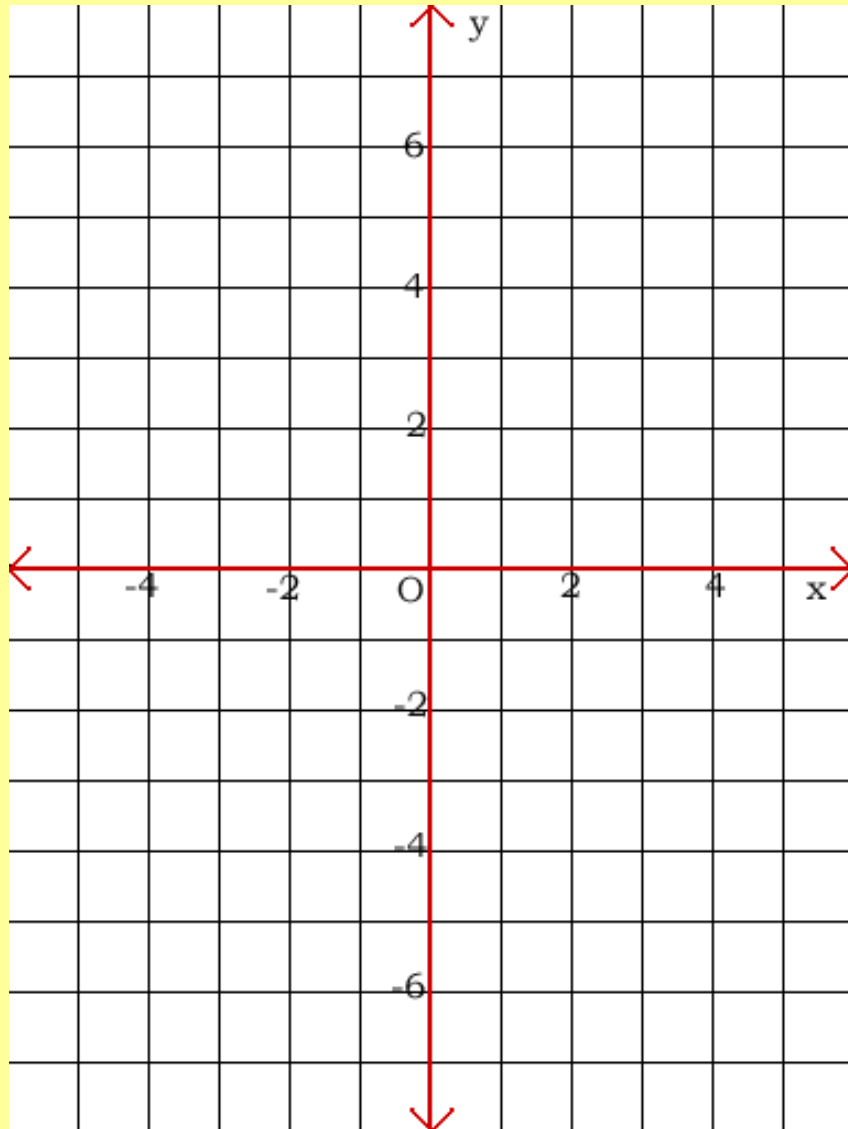


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Ex1 Continuity of a Function

Discuss the continuity of each function



$$y = \sin(x)$$

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One-sided Limits

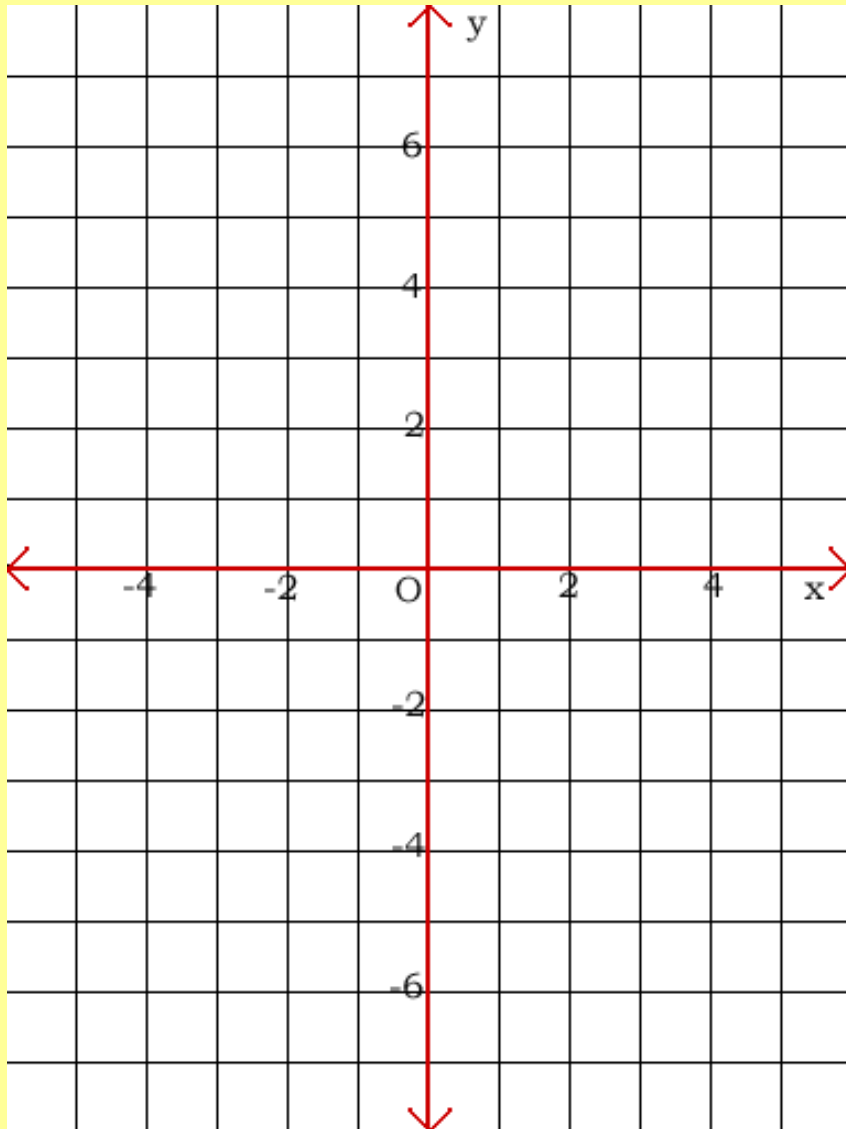
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Example:
One-sided limit
 x approaches c from the right

$$\lim_{x \rightarrow c^+} f(x) = L$$

From the direction of the
positive numbers



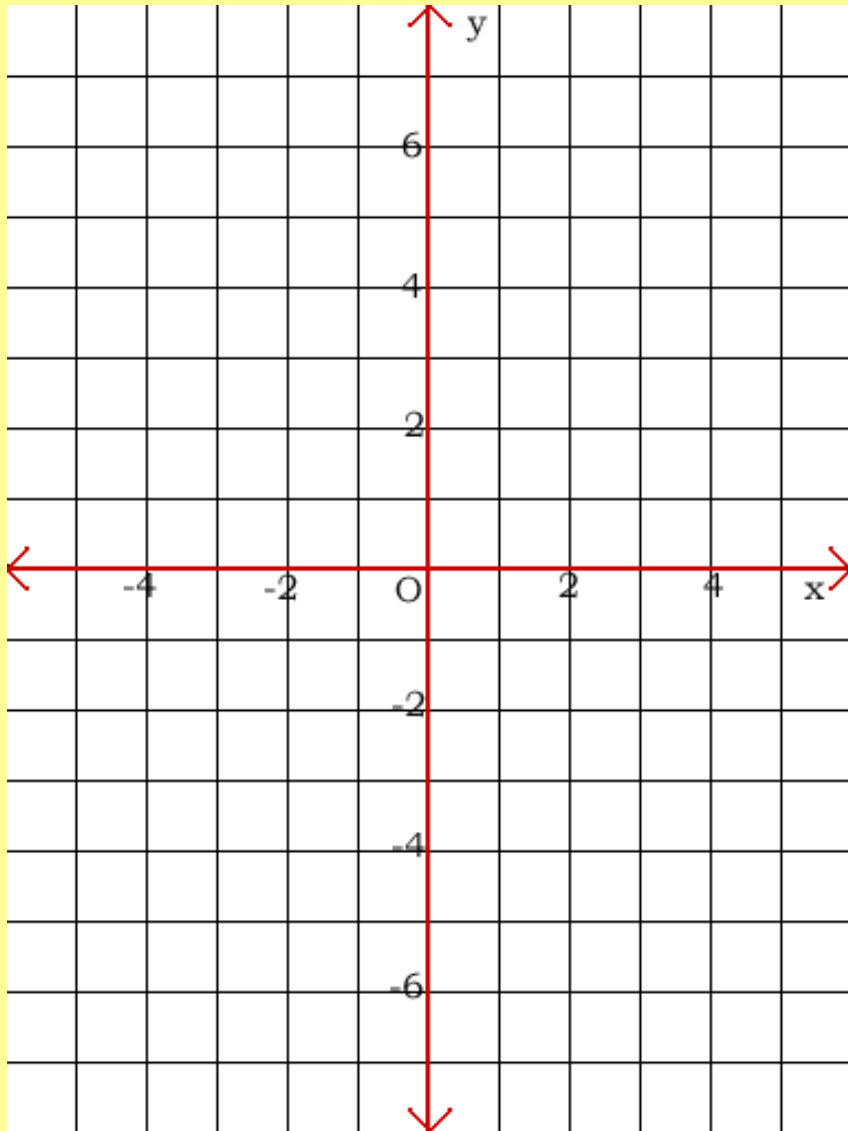
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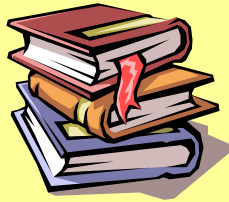
Example:
One-sided limit
 x approaches c from the left

$$\lim_{x \rightarrow c^-} f(x) = L$$

From the direction of the
negative numbers



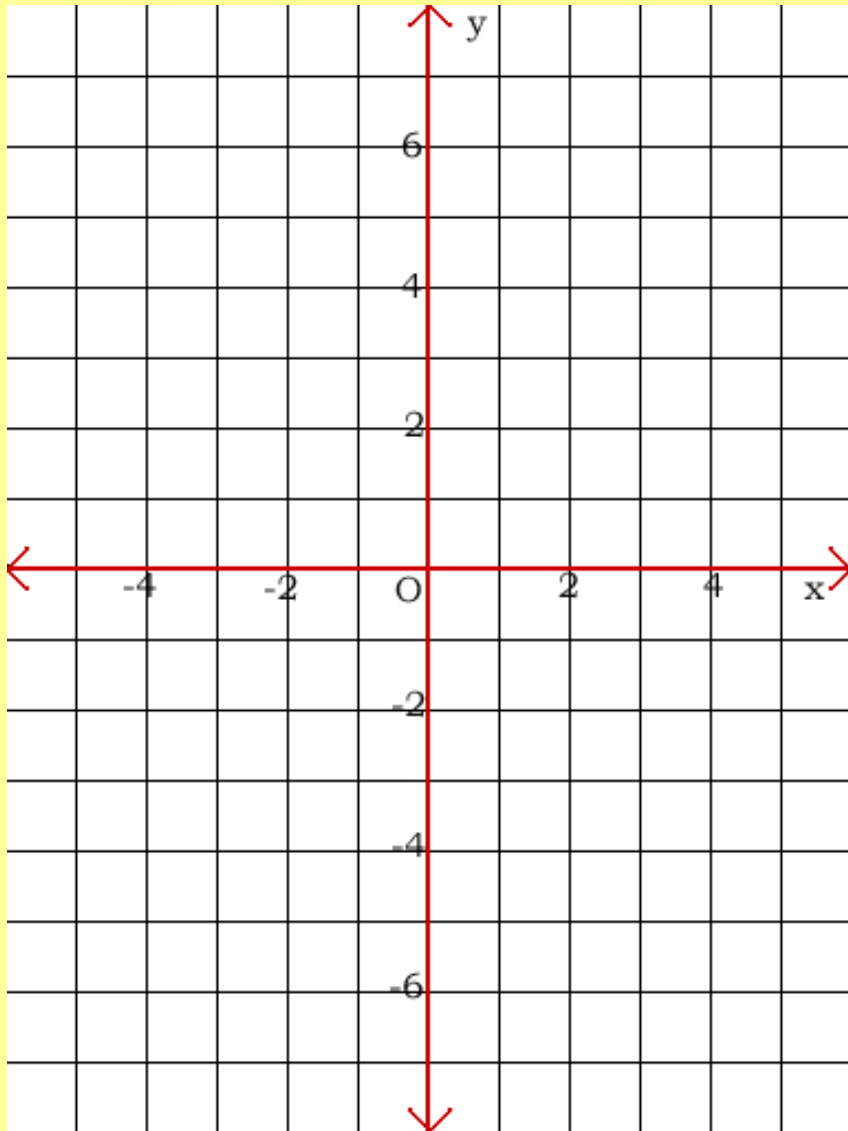
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Ex 2 One-sided limit

Find the limit

$$\lim_{x \rightarrow -2^+} \sqrt{4 - x^2}$$



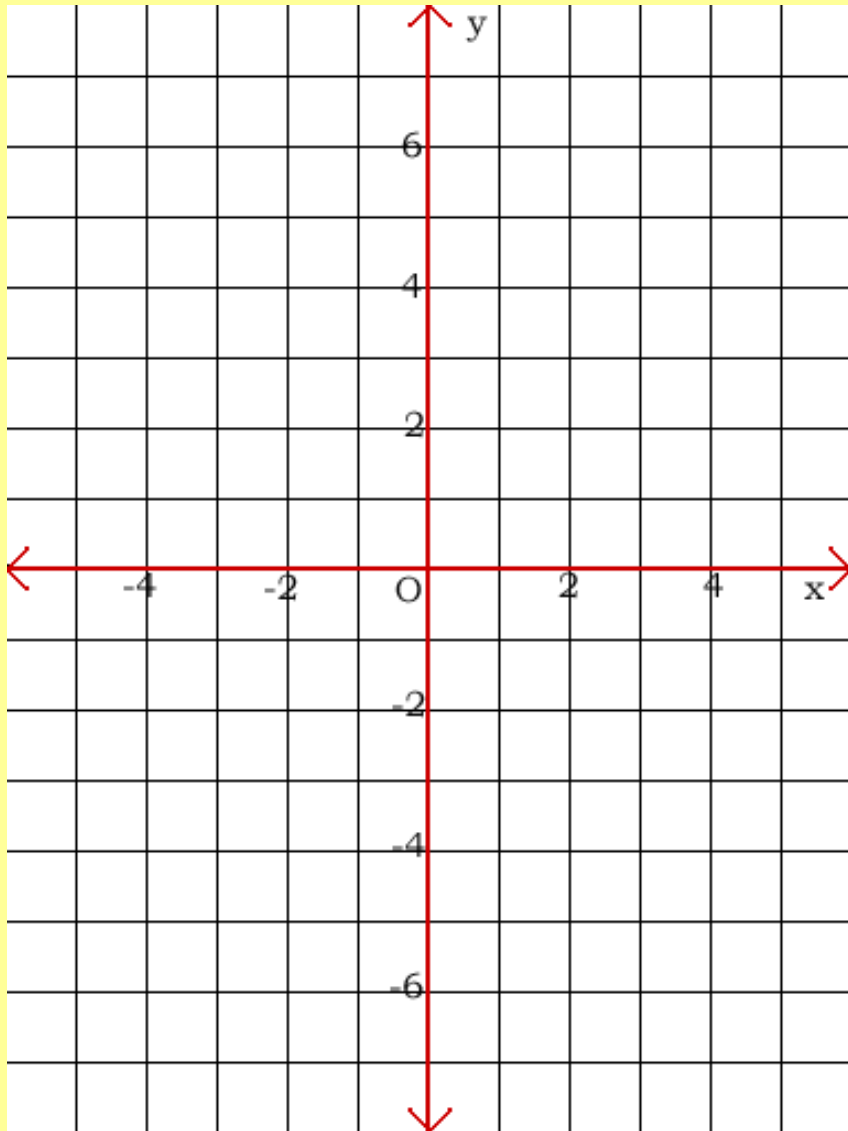
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Ex 2 One-sided limit

Find the limit

$$\lim_{x \rightarrow 2^-} \sqrt{4 - x^2}$$

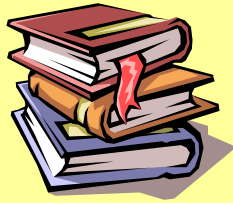


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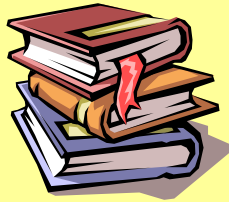
The Greatest Integer Function

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What is it?

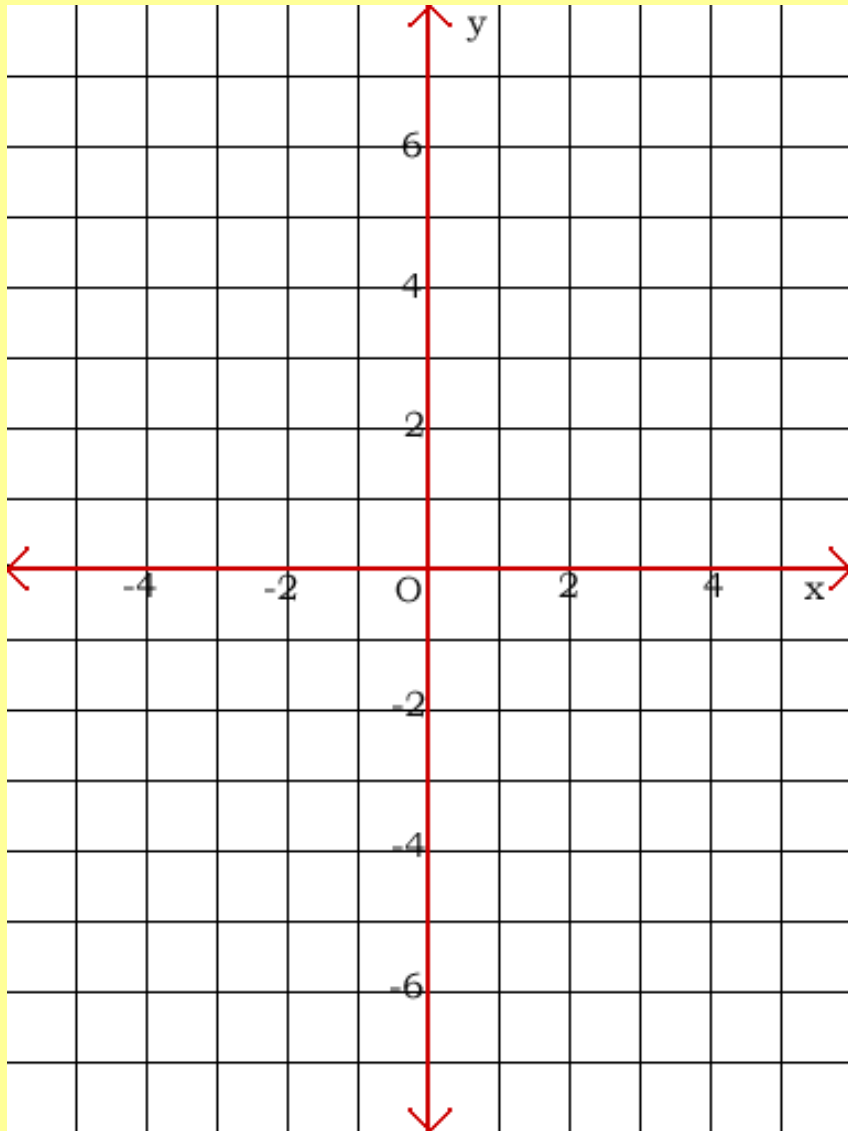
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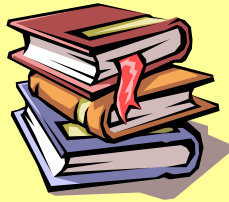
Ex 3 The Greatest Integer Function

Find the limit

$$\lim_{x \rightarrow 0^+} \lfloor x \rfloor$$



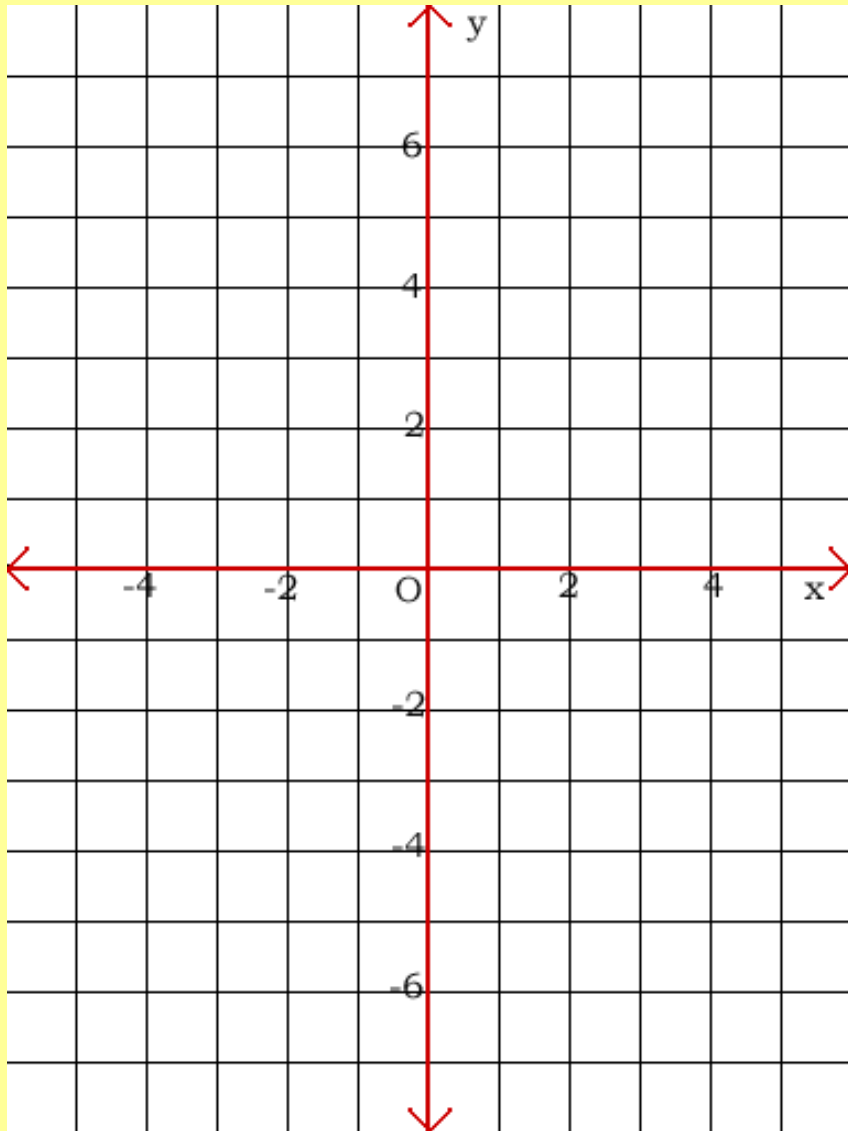
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Ex 3 The Greatest Integer Function

Find the limit

$$\lim_{x \rightarrow 0^-} \lceil x \rceil$$



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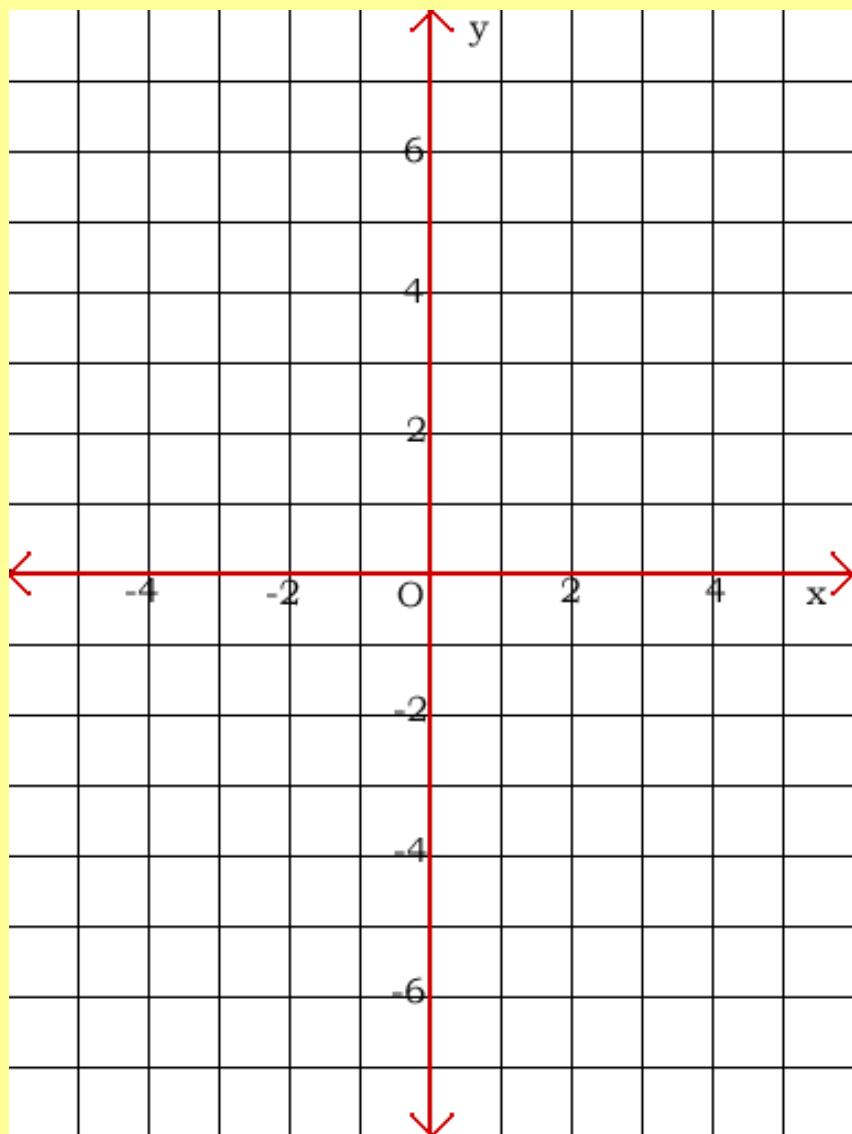
Ex 4 Continuity on a closed interval

Discuss the continuity of

$$f(x) = \sqrt{1-x^2}$$

$$\lim_{x \rightarrow -1^+} \sqrt{1-x^2} = 0 = f(-1)$$

$$\lim_{x \rightarrow -1^-} \sqrt{1-x^2} = 0 = f(1)$$



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What types of functions ARE continuous?

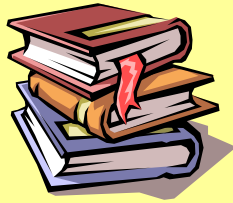
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Ex 6 Applying Properties of Continuity

$$f(x) = x + \sin(x)$$

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Ex 6 Applying Properties of Continuity

$$f(x) = 3 \tan(x)$$

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Ex 6 Applying Properties of Continuity

$$f(x) = \frac{x^2 + 1}{\cos x}$$

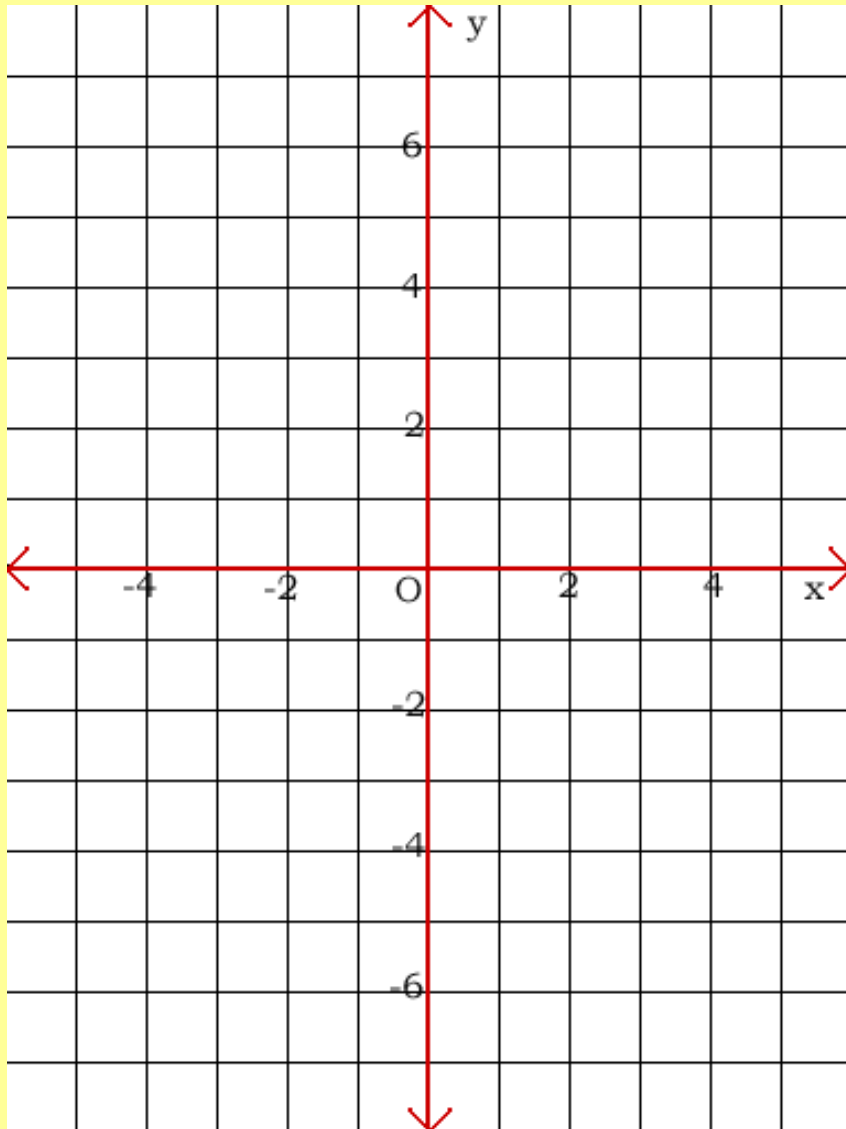
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Ex 7 Testing for Continuity

Describe the interval(s) on which each function is continuous

$$f(x) = \tan(x)$$

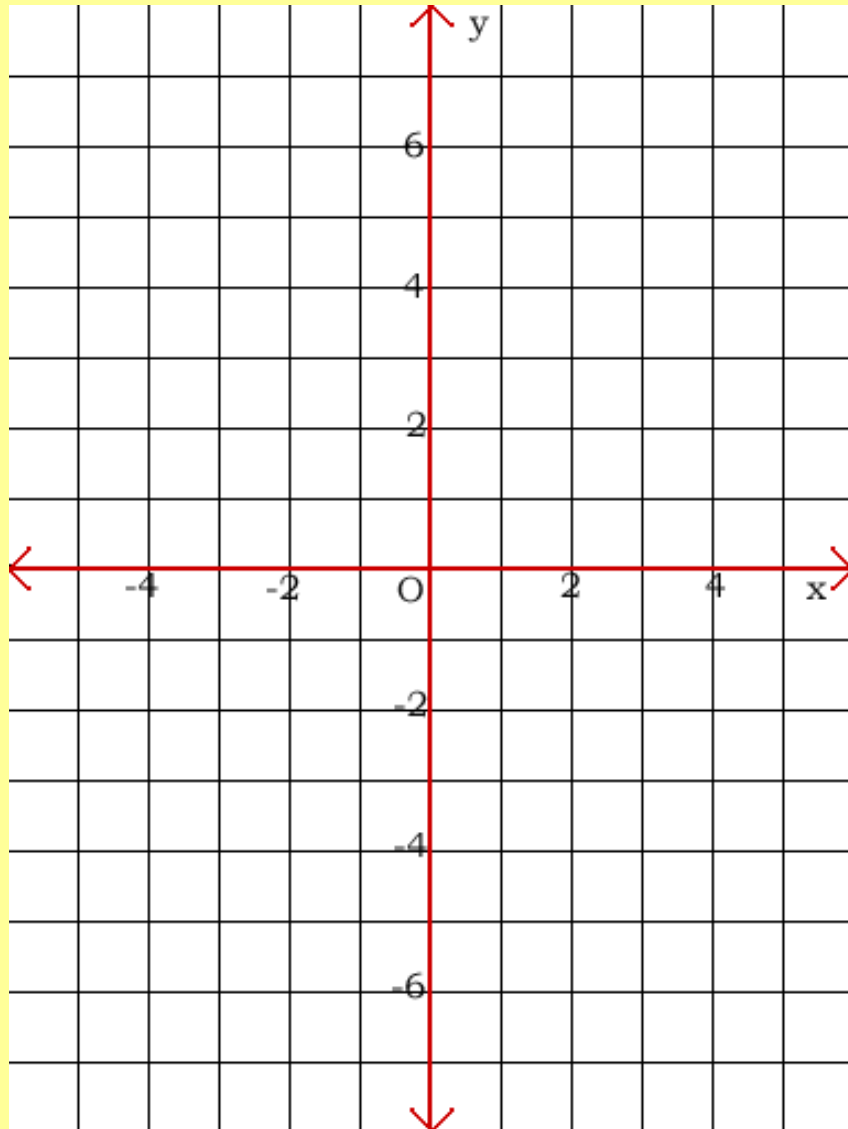


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Ex 7 Testing for Continuity

Describe the interval(s) on which each function is continuous



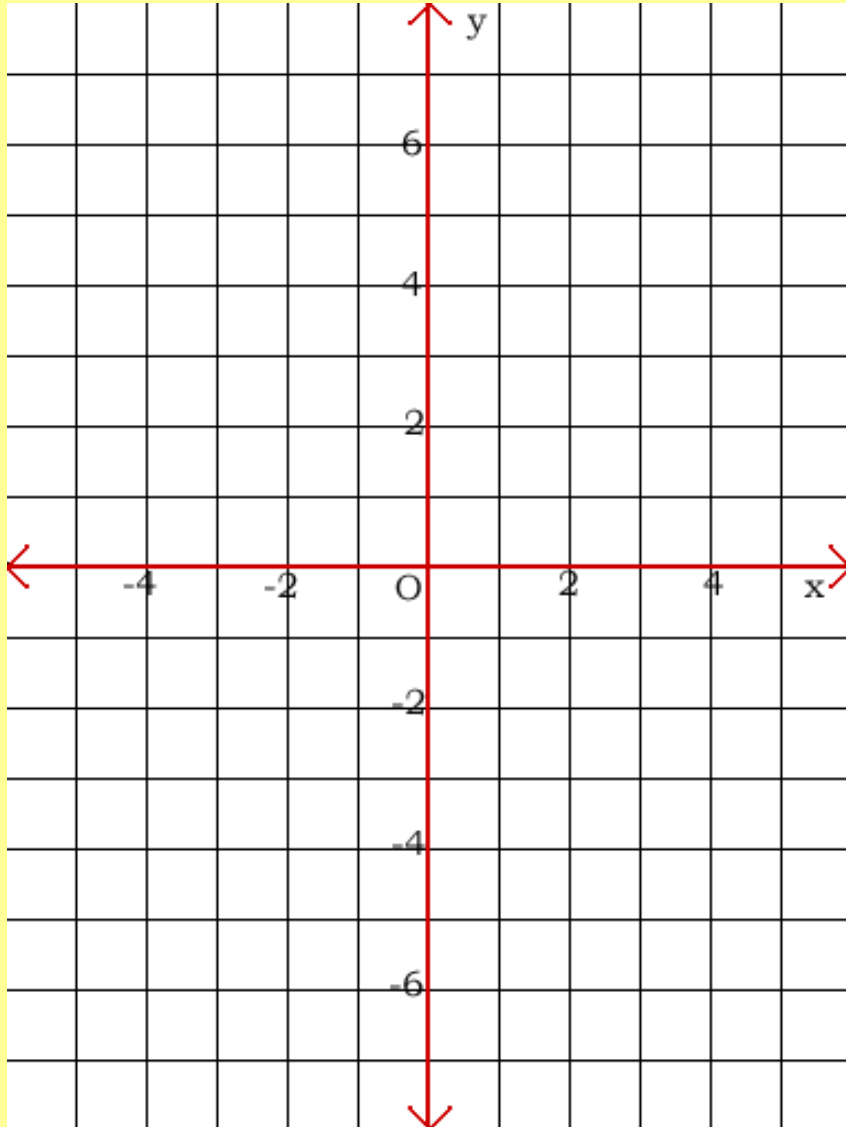
$$g(x) = \left\{ \begin{array}{l} \sin \frac{1}{x}, x \neq 0 \\ 0, x = 0 \end{array} \right\}$$

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Non-Removable Continuity

What does that look like graphically and algebraically



$$f(x) = \frac{x}{x^2 - 1}$$

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Continuity and One-sided Limits



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The best way to forget your own problems is to help someone else solve theirs.





You Can Do This!