

Calculus for Everyone

Please contact Mary.Lester@arkansas.gov,
if you would like the original PowerPoint slides

Finding a Limit: Dividing out and Rationalizing



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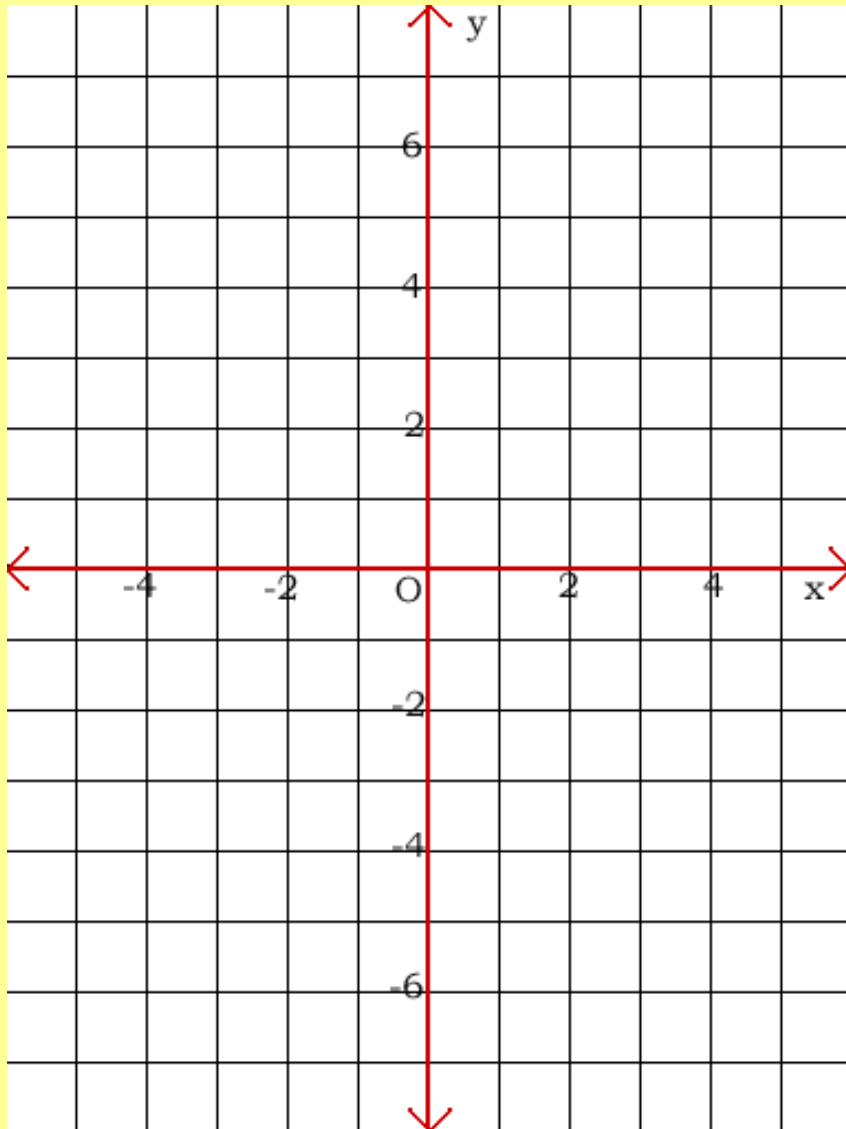
**These are template
slides meant to be filled
out by teachers and
students**



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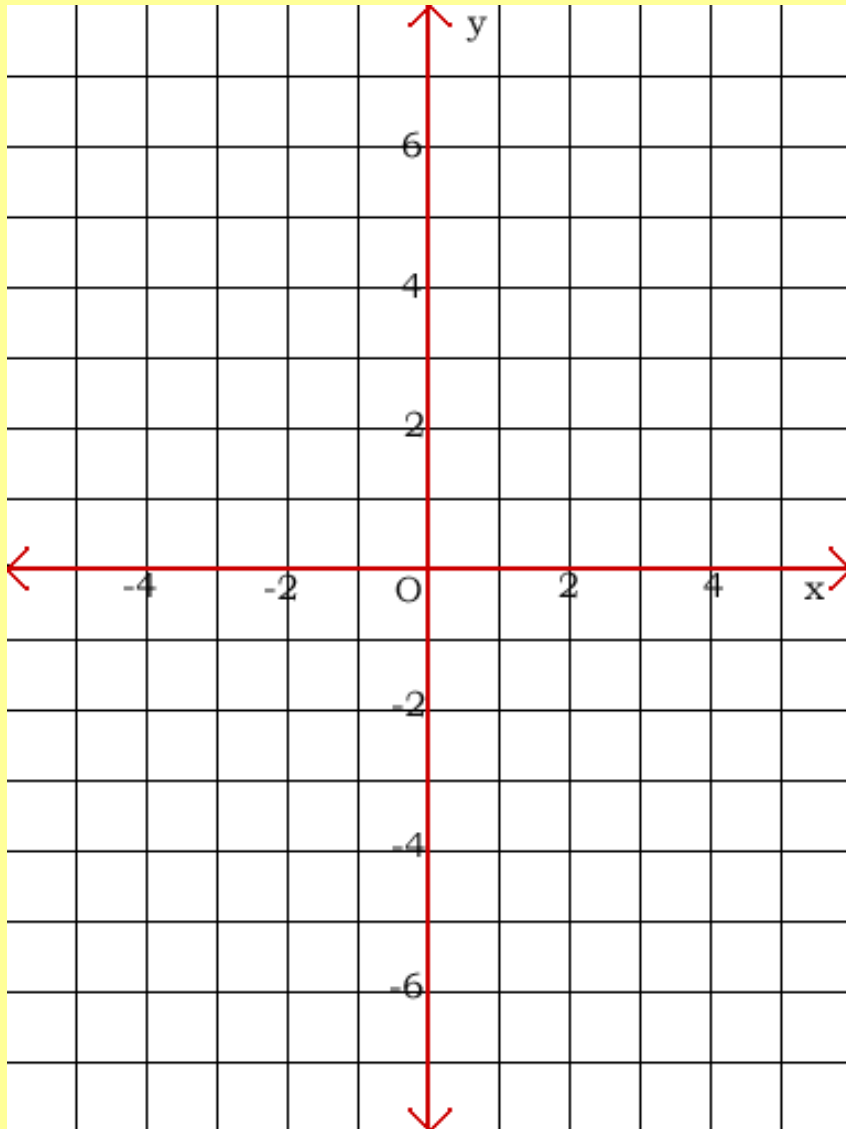
A little limit review...

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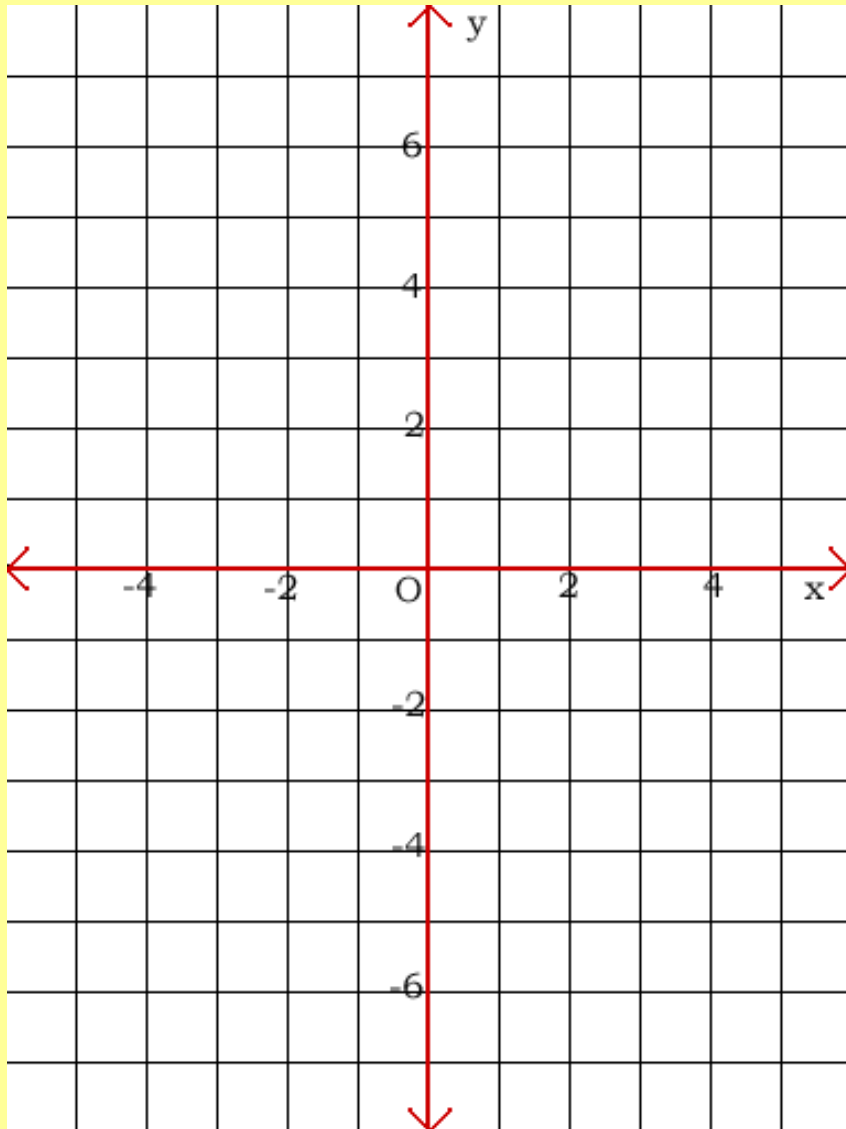
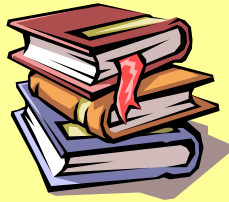
What is a limit?

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When does a limit occur?

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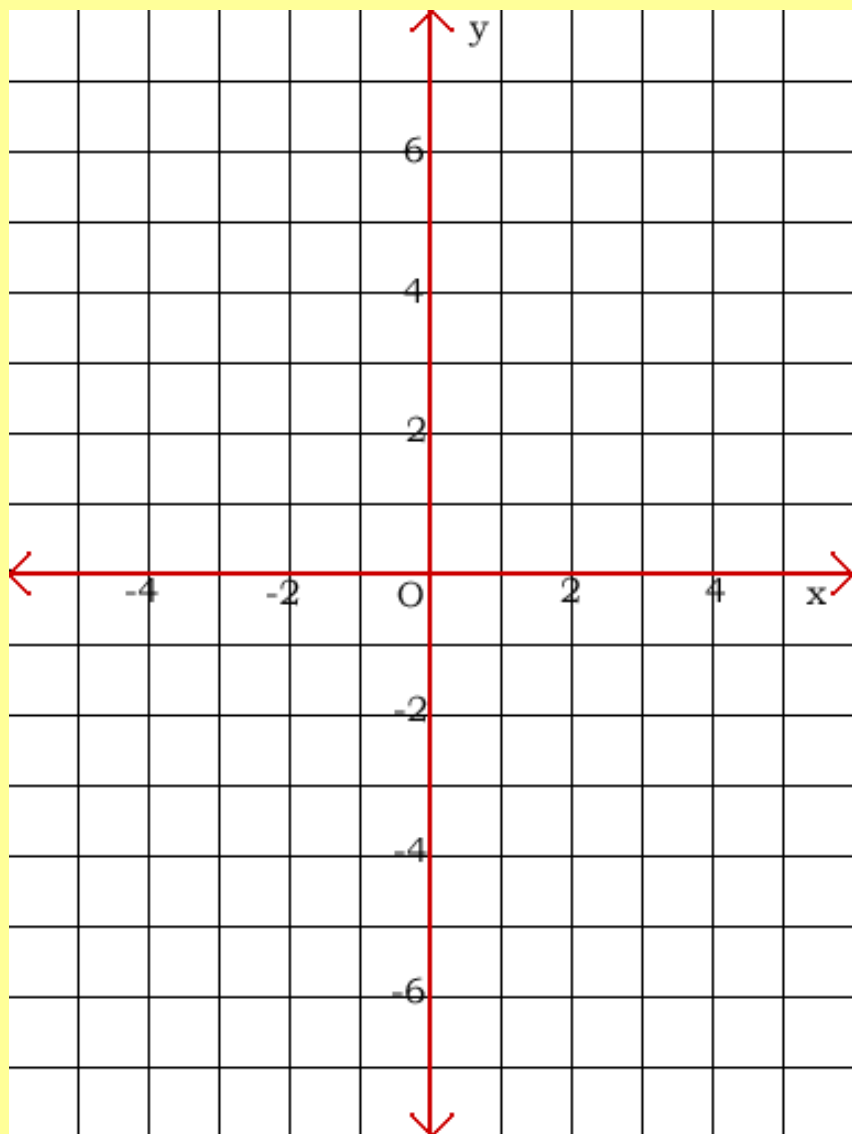


Does the functional value
have to equal the limit?

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**When does a limit
NOT occur?**

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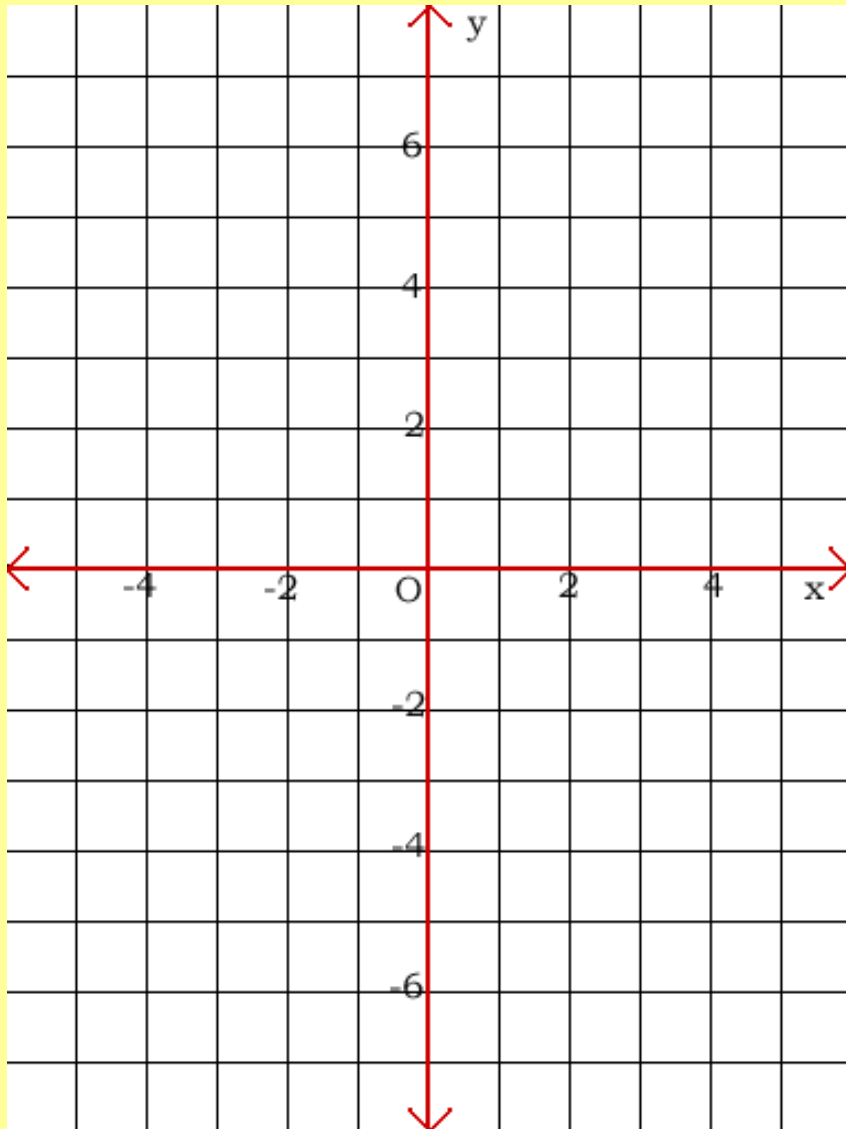
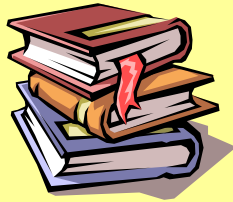
When does a limit **not** occur?

Doesn't approach the same value from the left and the right

(A Break)

What does that look like?

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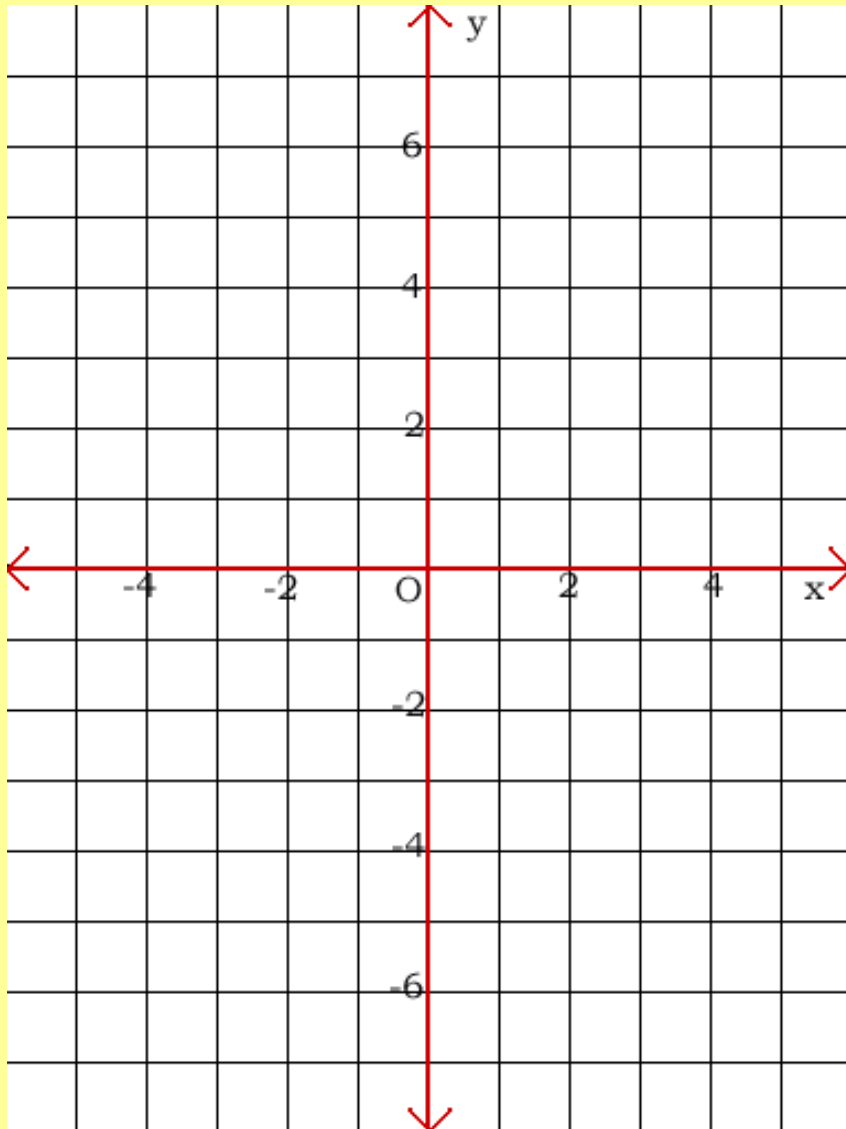
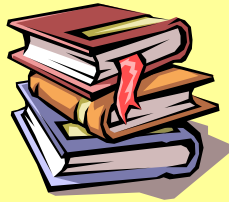


When does a limit **not** occur?

Unbounded behavior

What does that look like?

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When does a limit **not** occur?

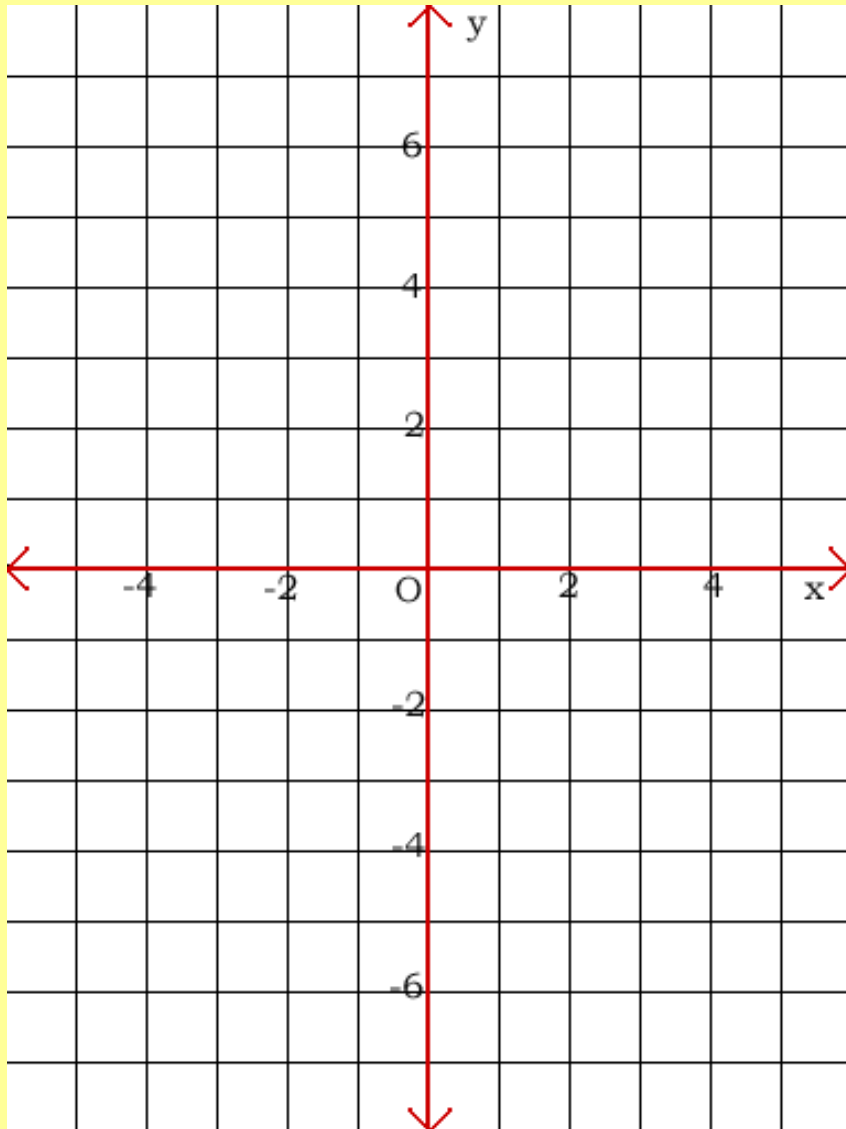
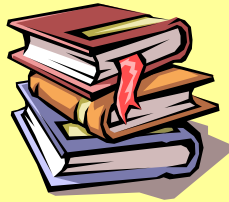
Oscillating Behavior

What does that look like?

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**How can we find out
what a limit value is?**

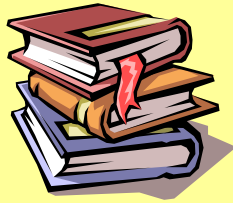
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What methods do we have of finding a limit?

Graphical

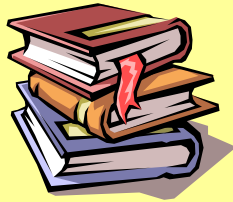
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What methods do we have of finding a limit?

Numerical

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What methods do we have of finding a limit?

Analytical---we will look at algebraic techniques today---and that is what I meant by analytical

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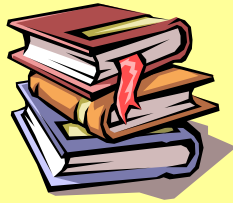
If functions are well-behaved, no holes, no gaps and you can draw them without picking up your pencil, they are called **continuous**

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And now the

Limit Theorems

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Limit of a constant is a constant

$$\lim_{x \rightarrow c} b = b$$

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Limit of a single variable power

$$\lim_{x \rightarrow c} x = c$$

“direct substitution”

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Limit of a variable raised to a power

$$\lim_{x \rightarrow c} x^n = c$$

“direct substitution”

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**Practice with
examples using those
properties**

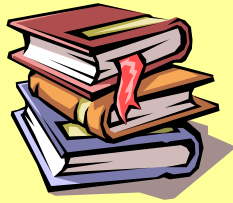
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Ex 1 Evaluating Basic Limits

a. $\lim_{x \rightarrow 2} 3 = 3$

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Ex 1 Evaluating Basic Limits

a. $\lim_{x \rightarrow 2} 3 = 3$

The limit of a constant is a constant

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Ex 1 Evaluating Basic Limits

b. $\lim_{x \rightarrow -4} x = -4$

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Ex 1 Evaluating Basic Limits

b. $\lim_{x \rightarrow -4} x = -4$

direct substitution

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Ex 1 Evaluating Basic Limits

C. $\lim_{x \rightarrow 2} x^2 = (2)^2 = 4$

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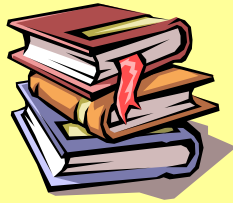


Ex 1 Evaluating Basic Limits

C. $\lim_{x \rightarrow 2} x^2 = (2)^2 = 4$

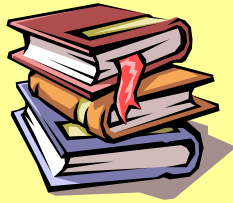
direct substitution

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**Don't go on calculus
"freak-out"---there are
a lot of symbols---
see the big
picture...You are going
to get this....**

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Properties of Limits

$$\lim_{x \rightarrow c} = L$$

and

$$\lim_{x \rightarrow c} = K$$

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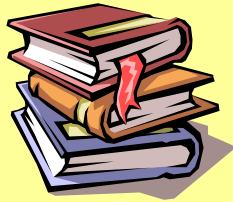


Scalar multiple

$$\lim_{x \rightarrow c} [bf(x)] = bL$$

What does the word **scalar** mean?

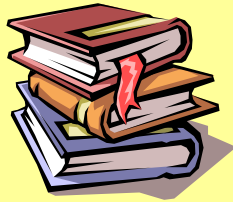
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Sum or difference

$$\lim_{x \rightarrow c} [f(x) \pm g(x)] = L \pm K$$

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Product

$$\lim_{x \rightarrow c} [f(x) \cdot g(x)] = L \cdot K$$

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Quotient

$$\lim_{x \rightarrow c} \left[\frac{f(x)}{g(x)} \right] = \frac{L}{K}$$

$$K \neq 0$$

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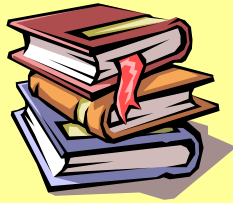
Power

$$\lim_{x \rightarrow c} [f(x)]^n = L^n$$

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**Practice with
examples using those
properties**

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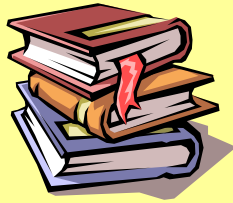
Ex 2 The limit of a polynomial

$$\lim_{x \rightarrow 2} 4x^2 + \lim_{x \rightarrow 2} 3$$

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**Direct substitution is
valid for any function
without domain
issues at the value(s)
in question**

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Ex 3 The limit of a rational function

$$\lim_{x \rightarrow 2} \frac{x^2 + x + 2}{x + 1}$$

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Limit of a function involving a radical

$$\lim_{x \rightarrow c} \sqrt[n]{x} = \sqrt[n]{c}$$

direct substitution

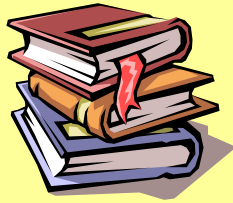
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Limit of a composite function

$$\lim_{x \rightarrow c} f(g(x)) = f(\lim_{x \rightarrow c} g(x)) = f(L)$$

direct substitution and with composite functions always work from the inside out

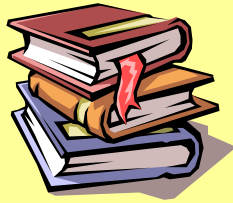
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Ex 4 The limit of a composite function

$$\lim_{x \rightarrow 2} \sqrt{x^2 + 4}$$

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Ex 4 The limit of a composite function

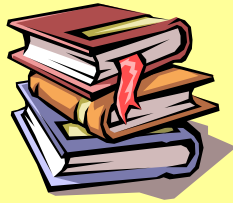
$$\lim_{x \rightarrow 3} \sqrt[3]{2x^2 - 10}$$

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The Limits of Trig Functions

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The limit of a trig function is simply the value of the function at that indicated input

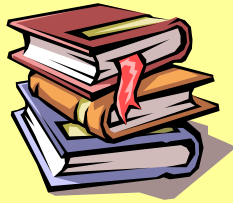
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And what does that mean????....

stay tuned!

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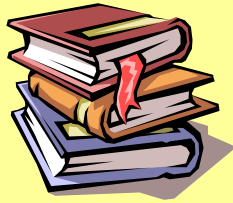


Ex 5 Limits of Trig Functions

$$\lim_{x \rightarrow 0} \tan x$$

direct substitution

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Ex 5 Limits of Trig Functions

$$\lim_{x \rightarrow \pi} (x \cos x)$$

direct substitution

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Ex 5 Limits of Trig Functions

$$\lim_{x \rightarrow 0} \sin^2(x)$$

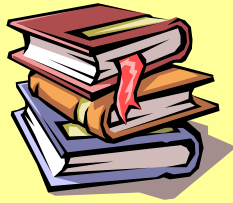
direct substitution

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Functions that agree at all but one point

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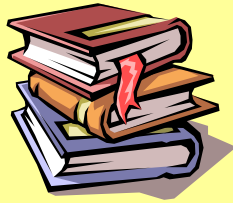


What is the formula for the sum and difference for the difference of two CUBES not squares?

$$(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$$

$$(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$$

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Ex 6 Finding the limit of a function

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$$

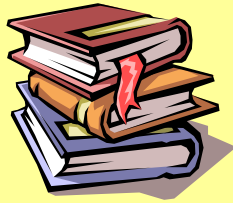
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Strategies for Finding limits

- 1) Direct substitution if possible
- 2) If not, find a function that agrees at all places but that one point
- 3) Use a graph or table to reinforce

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**Functions that differ only at one point
Technology Alert!**

May not look like there is a hole
there but there is....

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Another algebra technique

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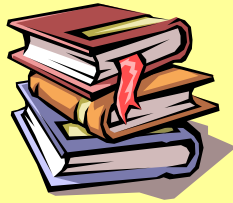


Ex 7 Dividing out technique

$$\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x + 3}$$

What if we use direct substitution first?

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Ex 7 Dividing out technique

$$\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x + 3}$$

We get the indeterminate form

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Ex 7 Dividing out technique

$$\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x + 3}$$

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***** Analytical (algebraic techniques) are what we added today---**

Strategies for Finding limits

- 1) Direct substitution if possible*****
- 2) If not, find a function that agrees at all places but that one point**
- 3) Use a graph or table to reinforce**

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**Where you go
and what you do
advertises
what you are**





You Can Do This !