Ar. Frameworks NPO1.4, NPO1.5, PAF1.2, PAF2.1

1) Two matrices can only be multiplied if the first matrix has the same number of columns as the second matrix has rows. In other words, the inside dimensions of the two matrices match:

   a) 2x2 and 2x4: these two can be multiplied, since the inside dimensions are both 2

   $$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f & g & h \\ i & j & k & l \end{bmatrix}$$

   b) 1x2 and 1x3: these two can't be multiplied, the inside dimensions are 2 and 1

   $$\begin{bmatrix} a & b \end{bmatrix} \quad \begin{bmatrix} c & d & e \end{bmatrix}$$

2) If two matrices can be multiplied, the outside dimensions will tell you the dimensions of the solution matrix:

   a) 2x2 and 2x4: multiplying these two matrices will result in a 2x4 matrix

   $$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 5 & 6 & 7 & 8 \\ 9 & 0 & 1 & 2 \end{bmatrix} \qquad \begin{bmatrix} 23 & 6 & 9 & 12 \\ 51 & 18 & 25 & 32 \end{bmatrix}$$

   b) 1x3 and 3x1: multiplying these two will result in a 1x1 matrix

   $$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \qquad \begin{bmatrix} 32 \end{bmatrix}$$

3) Once you've determined if two matrices can be multiplied, and what size the solution matrix will be, you will begin multiplying. This is where the importance of the columns matching the rows becomes obvious.

   a) We multiply matrices by multiplying the Rows of the first matrix by the Columns of the second matrix and adding the products:

      i) $$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \cdot \begin{bmatrix} g & h \\ i & j \end{bmatrix} \qquad \begin{bmatrix} a \cdot g + b \cdot i & a \cdot h + b \cdot j \\ c \cdot g + d \cdot i & c \cdot h + d \cdot j \\ e \cdot g + f \cdot i & e \cdot h + f \cdot j \end{bmatrix}$$

      ii) In this example, we multiply [*a b*] (first Row of first matrix) times [*g i*] first Column of second matrix) by multiplying *a* times *g* and *b* times *i*, then adding the products. This results in the value in Row 1, Column 1 of the solution matrix (above).

iii) **For this to work, there must be the same number of items in the rows of matrix 1 and the columns of matrix 2.**

iv) Continue this pattern until each Row of the first matrix has been multiplied times each Column of the second matrix.

b) Now, for an example with actual values, we have Matrix A and Matrix B. Since these matrices are a 3x<u>2</u> and a <u>2</u>x2, we can multiply A*B. ORDER IS IMPORTANT HERE! We could not multiply B*A, since that would be a 2x<u>2</u> times a <u>3</u>x2, and the inside dimensions don't match. AB will be a 3x2 matrix.

$$\begin{bmatrix} -2 & 3 \\ 1 & -4 \\ 6 & 0 \end{bmatrix} = A \qquad \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix} = B$$

c) We always multiply the ROWS of A times the COLUMNS of B and add our products.

Start with Row 1 [-2, 3] of A and Column 1 [-1, -2] of B:

i) (-2 * (-1)) + (3 * (-2)) = -4 – this value goes in Row 1, Column 1 of the solution matrix.

Then do Row 1 of A times Column 2 [3, 4] of B:

ii) (-2 * 3) + (3 * 4) = 12 – this value is the entry for Row 1, Column 2

Continue this pattern for Row 2 of matrix A…

iii) (1 * (-1)) + (-4 * (-2)) = 7 – this is Row 2, Column 1

iv) (1 * 3) + (-4 * 4) = -13 – this is Row 2, Column 2

And Row 3 of matrix A…

v) (6 * (-1)) + (0 * (-2)) = -6 – this is Row 3, Column 1

vi) (6 * 3) + (0 * 4) = 18 – this is Row 3, Column 2

d) The solution matrix is:

$$\begin{bmatrix} -4 & 6 \\ 7 & -13 \\ -6 & 18 \end{bmatrix}$$

4) Properties of Matrix Multiplication (let A, B and C be matrices and let *c* be a scalar)
   a) Associative Property of Matrix Multiplication
      i) A(BC) = (AB)C
   b) Associative Property of Scalar Multiplication
      i) *c*(AB) = (cA)B = A(cB)
   c) Left Distributive Property
      i) A(B + C) = AB + BC
   d) Right Distributive Property
      i) (A + B)C = AC + BC

5) **ORDER IS VERY, VERY IMPORTANT IN MATRIX MULTIPLICATION**